UNIT – 2
MOMENT OF INERTIA

2.1 CENTRE OF GRAVITY

Centre of gravity of a body can be defined as the point through which resultant of force of gravity or weight of the body acts. A body is having only one centre of gravity for all positions of the body. It is normally denoted by *G* or *CG* or *g* or *cg*.

2.2 CENTROID

Plane figures such as square, rectangle, circle, triangle, quadrilateral, etc., have only areas, but no mass. The point at which the total area of a plane figure is assumed to be concentrated is known as the centroid of that area (or) the centre of area of such figures is known as centroid. The centroid is denoted by *G* or *CG* or *g* or *cg*. The centroid and centre of gravity are at the same point.

2.3 DIFFERENCE BETWEEN CENTRE OF GRAVITY AND CENTROID

The following are the differences between centre of gravity and centroid :

(i) The term centre of gravity applies to bodies with mass and weight, and centroid applies to plane areas.

(ii) Centre of gravity of a body is a point through which the resultant gravitational force or weight acts for any orientation of the body, whereas centroid is a point in a plane area such that the moment of area about any axis through that point is zero.

2.4 METHODS OF FINDING CENTRE OF GRAVITY OF SIMPLE FIGURES

The following are the methods to find out the centre of gravity of simple figures.

(i) By geometrical considerations

(ii) By moments

(iii) By graphical method

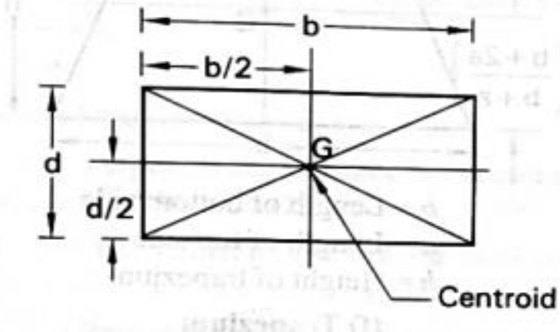
2.5 CENTRE OF GRAVITY BY GEOMETRIC CONSIDERATIONS

CG of Rectangle, Triangle, Circle, Semi-circle, Trapezium and Cone

Centre of gravity of simple figures such as square, rectangle, triangle, circle, trapezium etc., can be found out from the geometry of the figure.

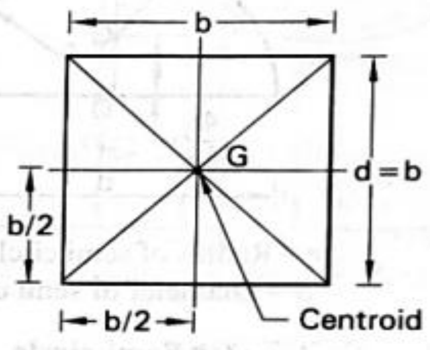
(i) CG of Rectangle

The centre of gravity (*G*) of a rectangle is at the point, where its diagonals meet each other. It is also the point of intersection of the lines joining the middle points of the opposite sides. It is shown in Fig. 2.1(a).



b = Length of longer-side ; d = Length of shorter side

(a) Rectangle



$b = d$ = Length of side

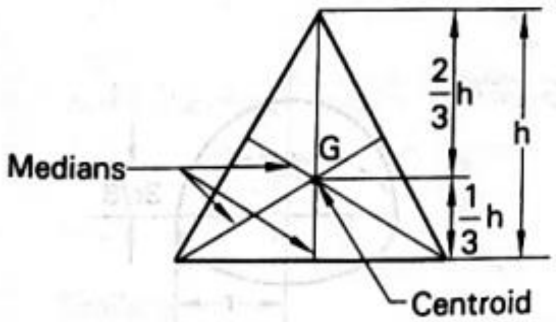
(b) Square

(ii) CG of Square

The centre of gravity (G) of a square is at the point, where its diagonals meet each other. It is also the point of intersection of the lines joining the middle points of the opposite sides. It is shown in Fig. 2.1(b).

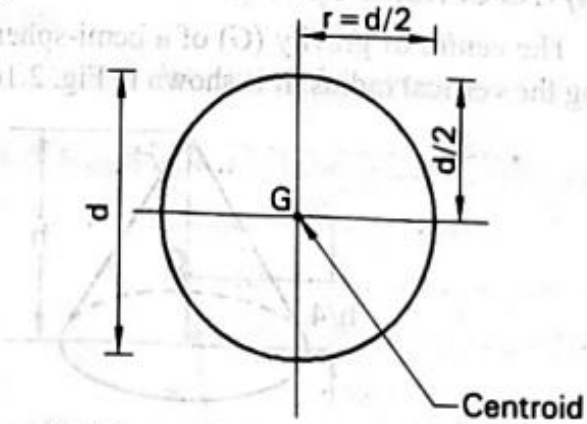
(iii) CG of Triangle

The centre of gravity(G) of a triangle lies at the point where the three medians of the triangle meet. It is shown in Fig. 2.1(c). The line connecting the vertex and the middle point of the opposite side of a triangle is known as median of the triangle.



h = Height of triangle

(c) Triangle



d = Diameter of circle ; r = Radius of circle

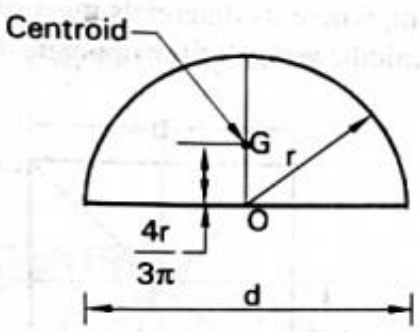
(d) Circle

(iv) CG of Circle

The centre of gravity(G) of a circle lies at its centre. It is shown in Fig. 2.1(d).

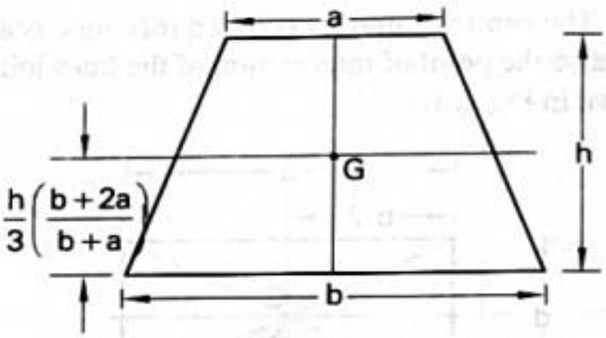
(v) CG of Semi-circle

The centre of gravity (G) of a semi-circle is at a distance of $4r/3\pi$ from its base, measured along the vertical radius. It is shown in Fig. 2.1(e).



r = Radius of semi circle
d = Diameter of semi circle

(e) Semi-circle



b = Length of bottom side
a = Length of top side
h = Height of trapezium

(f) Trapezium

(vi) CG of Trapezium

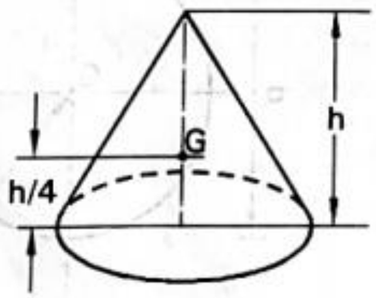
The centre of gravity(G) of a trapezium with parallel side 'a' and 'b' is at a distance of $\left[\frac{h}{3}\times\left(\frac{b+2a}{b+a}\right)\right]$ from the bottom side 'b'. h is the height of trapezium or the perpendicular distance between the two parallel sides. It is shown in Fig. 2.1(f).

(vii) CG of Cone

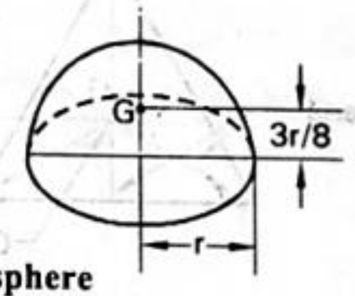
The centre of gravity (G) of a right circular solid cone is at a distance of $h/4$ from its base, measured along the vertical axis. It is shown in Fig. 2.1(g).

(viii) CG of hemi-sphere

The centre of gravity (G) of a hemi-sphere is at a distance of $3r/8$ from its base, measured along the vertical radius. It is shown in Fig. 2.1(h).



(g) Cone h = Height of cone



(h) Hemi-sphere r = Radius of hemi-sphere

Fig. 2.1

2.6 CENTRE OF GRAVITY OF PLANE FIGURES BY THE METHOD OF MOMENTS

Plane geometrical figures such as T-section, I-section, L-section, etc., have only areas and their masses are negligible. Hence their weights are neglected while calculating the centre of gravity.

Consider a plane figure of total area 'A' whose centroid is to be determined. Divide the total area 'A' into number of small areas whose centroids are known as shown in Fig. 2.2. Let a_1, a_2, a_3,...., etc be the small areas and g_1, g_2, g_3,..... etc be their centroids respectively.

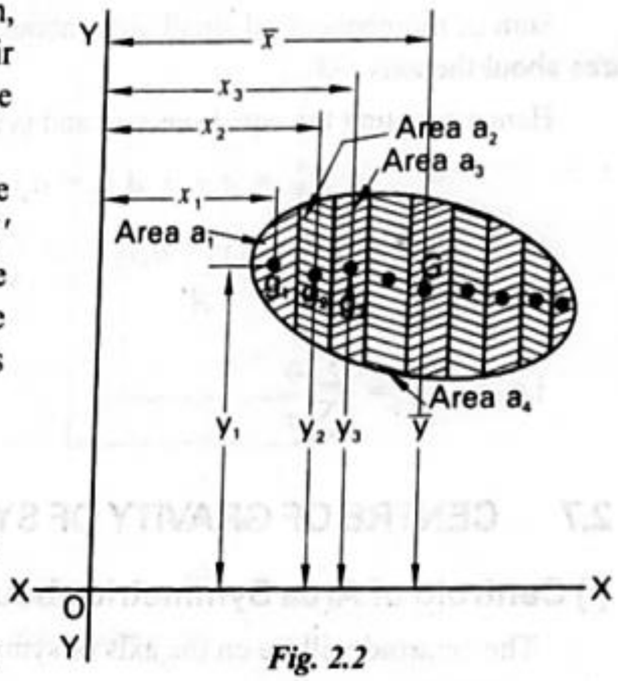


Fig. 2.2

Let,

Total area $A = a_1 + a_2 + a_3 + ...$

x_1 = Perpendicular distance between centroid of area a_1 and the axis OY

x_2 = Perpendicular distance between the centroid of area a_2 and the axis OY

x_3 = Perpendicular distance between the centroid of area a_3 and the axis OY and so on

y_1 = Perpendicular distance between the centroid of area a_1 and the axis OX

y_2 = Perpendicular distance between the centroid of area a_2 and the axis OX

y_3 = Perpendicular distance between the centroid of area a_3 and the axis OX and so on.

Let G is the centroid of the total area A whose perpendicular distance from the axis OY is $\bar{x}$ and from the axis OX is $\bar{y}$.

Sum of moments of all small areas about axis $OY = a_1x_1 + a_2x_2 + a_3x_3 + ... = \sum ax$ --- (i)

Moment of total area about axis $OY = A\bar{x}$ --- (ii)

Sum of moments of all small areas about the axis OY must be equal to the moment of total area about the axis OY.

Hence equating the equations (i) and (ii), we get

$$A\bar{x} = a_1x_1 + a_2x_2 + a_3x_3 + ...$$

$$\therefore \quad \bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 ......}{A}, \text{ where } A = a_1 + a_2 + a_3 + ... = \sum a \qquad \text{--- (iii)}$$

$$\text{i.e.,} \quad \bar{x} = \frac{\sum ax}{\sum a}$$

Similarly,

Sum of moment of all small areas about axis $OX = a_1y_1 + a_2y_2 + a_3y_3 + ... = \sum ay$ --- (iv)

Moment of total area about axis $OX = A\bar{y}$ --- (v)

Sum of moments of all small areas about the axis OX must be equal to the moment of total area about the axis OX.

Hence equating the equations (iv) and (v), we get,

$$A\bar{y} = a_1y_1 + a_2y_2 + a_3y_3 + ...$$

$$\therefore \quad \bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 ......}{A}, \text{ where } A = a_1 + a_2 + a_3 + ... = \sum a \qquad \text{--- (vi)}$$

$$\text{i.e.,} \quad \bar{y} = \frac{\sum ay}{\sum a}$$

2.7 CENTRE OF GRAVITY OF SYMMETRICAL SECTIONS

(i) Centroid of Area Symmetric about an Axis

The centroid will lie on the axis of symmetry, if a plane area has an axis of symmetry. Here the first moment of area about an axis of symmetry is zero. In such a case, only either $\bar{x}$ or $\bar{y}$ distance need to be calculated. In Fig. 2.3(a), x-axis is the axis of symmetry and hence centroid G will lie on that axis. Here only $\bar{x}$ distance has to be calculated, as centroid 'G' lies on the axis of symmetry. In Fig. 2.3(b), y-axis is the axis of symmetry and hence centroid 'G' will lie on that axis. Here only $\bar{y}$ distance has to be calculated, as centroid 'G', lies on the axis of symmetry.

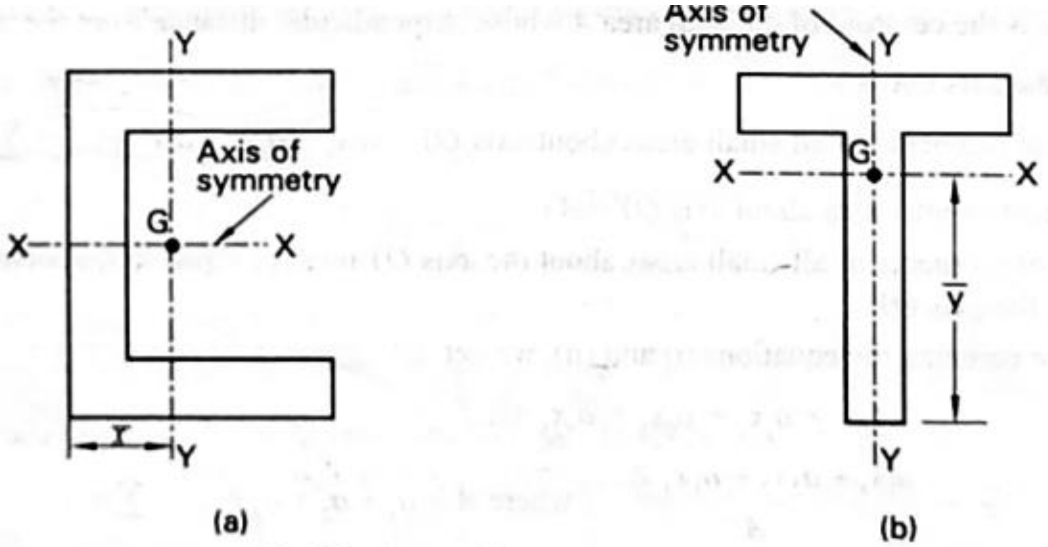
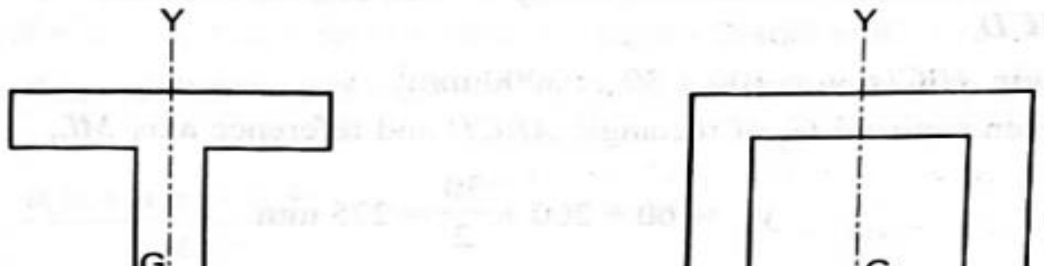


Fig. 2.3 : Area with one axis of symmetry

(ii) Centroid of Area Symmetric about Two Axes

The centroid G lies at the intersection of the two axes of symmetry, if a plane area has two axes of symmetry. The centroid 'G' of such an area can be located by inspection [Fig. 2.4(a) and (b)]. Here the first moment of area about both the axes will be zero.

AXIS OF REFERENCE



The centre of gravity (CG) of a body is always calculated with reference to some assumed axis known as Axis of Reference.

2.8 IMPORTANT POINTS TO REMEMBER

(i) The centre of gravity of a body is always calculated with reference to some assumed axis known as reference axis. Reference axis is the axis about which the moments of areas are taken.

(ii) For calculating $\bar{y}$, the bottom most line of the figure is taken as the reference axis.

(iii) Similarly, for calculating $\bar{x}$, the extreme left end line is taken as the reference axis.

(iv) If the section is symmetrical about $X-X$ axis, then only $\bar{x}$ distance has to be calculated, since centroid G lies on the axis of symmetry.

(v) Similarly, if the section is symmetrical about $Y-Y$ axis, then only $\bar{y}$ distance has to be calculated, since centroid G lies on the axis of symmetry.

PROBLEMS

P1

Determine the centroid of a 120 mm × 150 mm × 30 mm T-section.

Solution : The T-section is shown in Fig. 2.6 and it is symmetrical about $Y-Y$ axis. As the centroid G lies on the axis of symmetry, only $\bar{y}$ distance has to be calculated. The given T-section is split up into two rectangles $ABCD$ and $EFHI$ as shown in Fig. 2.6. G_1 and G_2 are the centroids of these two rectangles respectively. Let the bottom edge IH be the reference axis.

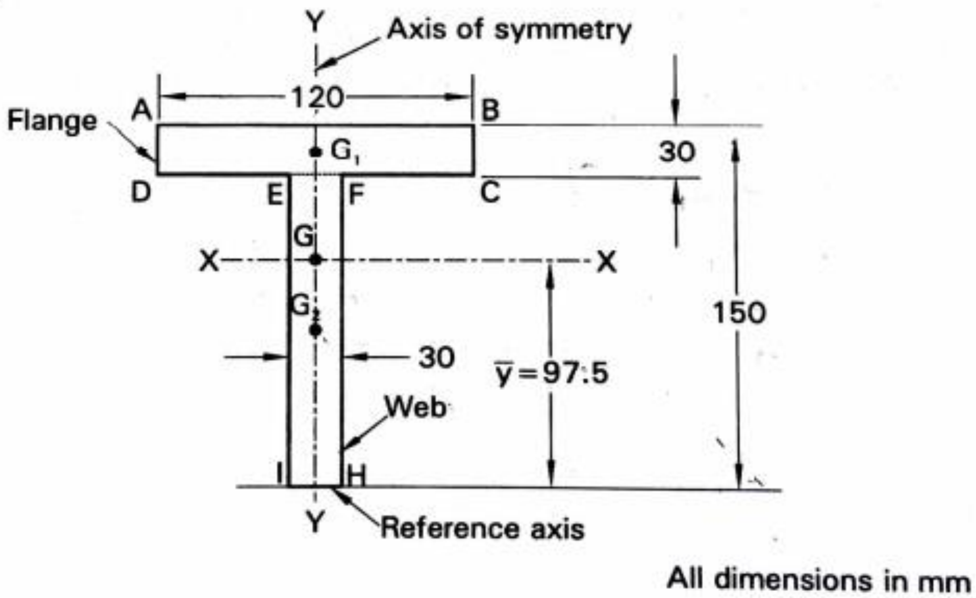


Fig. 2.6

(i) Flange ABCD

Area of rectangle $ABCD$, $a_1 = 120 \times 30 = 3600 \text{ mm}^2$

Distance between centroid G_1 of rectangle $ABCD$ and reference axis IH,

$$y_1 = 150 - \frac{30}{2} = 135 \text{ mm}$$

(ii) Web EFHI

Area of rectangle *EFHI*, $a_2 = (150 - 30) \times 30 = 3600\ \text{mm}^2$

Distance between centroid G_2 of rectangle *EFHI* and reference axis *IH*,

$$y_2 = \frac{150 - 30}{2} = 60\ \text{mm}$$

$$\text{Total area, } A = a_1 + a_2 = 3600 + 3600 = 7200\ \text{mm}^2$$

$$\bar{y} = \frac{\text{Sum of moments of individual areas about reference axis } IH}{\text{Total area}}$$

$$= \frac{a_1 y_1 + a_2 y_2}{A} = \frac{(3600 \times 135) + (3600 \times 60)}{7200} = 97.5\ \text{mm}$$

Therefore centroid *G* of the T-section is at a distance of 97.5 mm from reference axis *IH* (i.e., bottom edge *IH*) and lies on the $Y - Y$ axis.

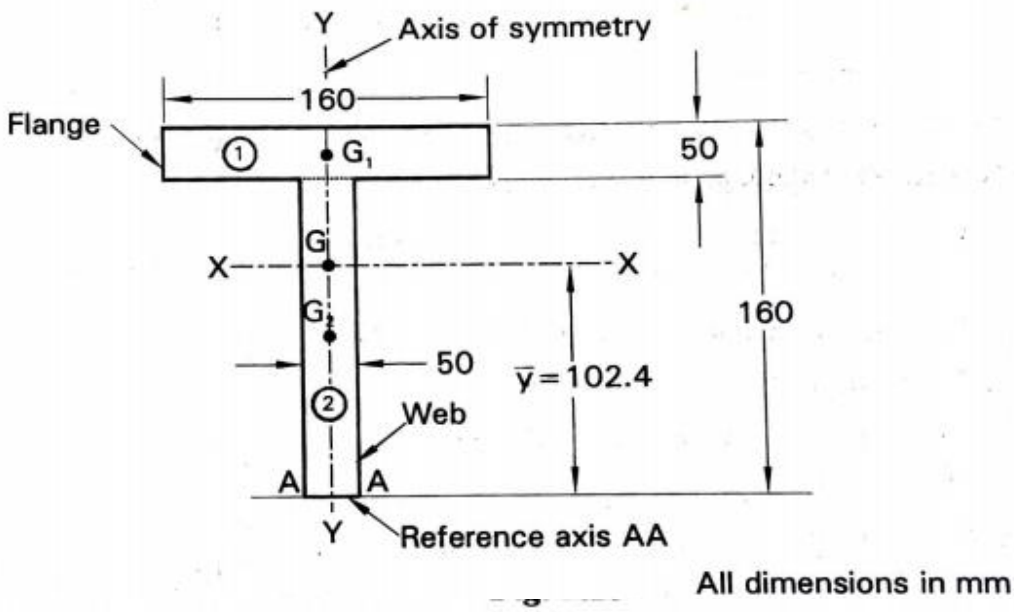
P2

Find the moment of inertia about the centroidal axis $X - X$ and $Y - Y$ of the T-section 160 mm wide and 160 mm deep. The flange and web thickness is 50 mm each.

Solution :

(i) Centroid (G) of the T-section

The T-section is shown in Fig. 2.25 and it is symmetrical about $Y - Y$ axis. As centroid *G* lies on the axis of symmetry, only $\bar{y}$ distance has to be calculated. The given T-section is split up into two rectangles (1) and (2) as shown in Fig. 2.25. Let the bottom edge *AA* of the web be the reference axis. G_1 and G_2 are the centroid of rectangles (1) and (2) respectively.



Rectangle (1)

Area of rectangle (1), $a_1 = 160 \times 50 = 8000\ \text{mm}^2$

Distance between G_1 and reference axis *AA*, $y_1 = 160 - \dfrac{50}{2} = 135\ \text{mm}$

Rectangle (2)

Area of rectangle (2), $a_2 = (160 - 50) \times 50 = 5500\ \text{mm}^2$

$\therefore$ Distance between centroid G of T-section and reference axis AA,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(8000 \times 135) + (5500 \times 55)}{8000 + 5500} = 102.4 \text{ mm}$$

P3

Find the centroid of a T section of size Flange 120 X 20 mm, and Web 120 X 20 mm.

Solution :

(i) Centroid of the T-section

The T-section is shown in Fig. 2.29 and it is symmetrical about $Y-Y$ axis. As centroid G lies on the axis of symmetry, only $\bar{y}$ distance has to be calculated. The given T-section is split up into two rectangles (1) and (2) as shown in Fig. 2.29. Let the bottom edge AA of the web be the reference axis. G_1 and G_2 are the centroid of rectangles (1) and (2) respectively.

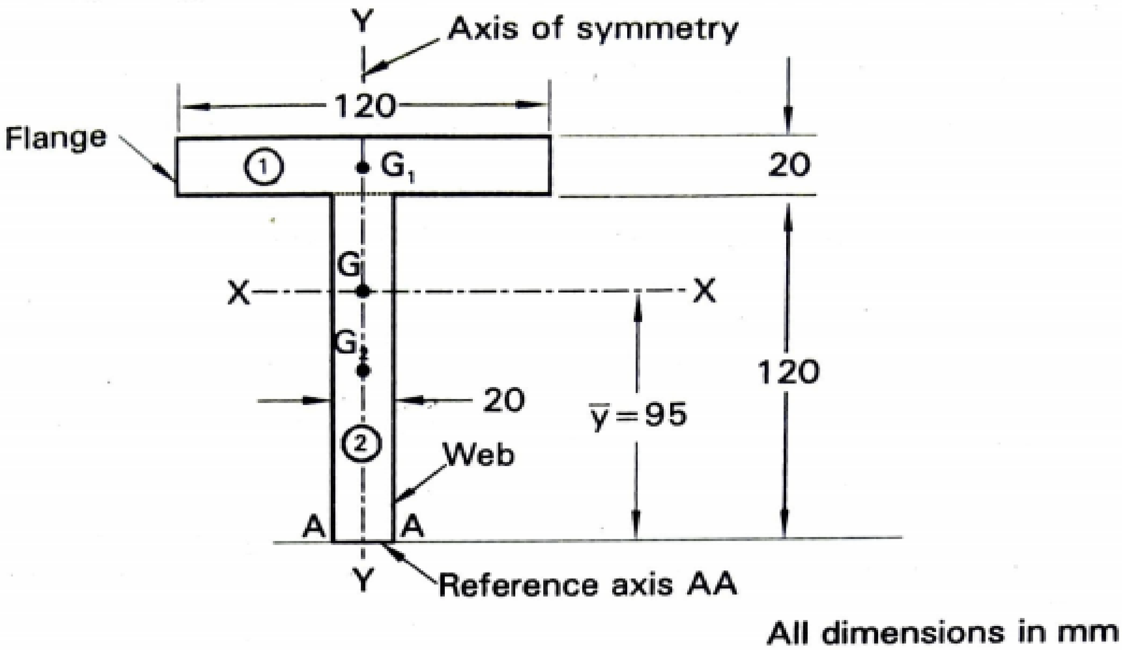


Fig. 2.29

Rectangle (1) :

Area of rectangle (1), $a_1 = 120 \times 20 = 2400 \text{ mm}^2$

Distance between G_1 and reference axis AA, $y_1 = 120 + \dfrac{20}{2} = 130$ mm

Rectangle (2) :

Area of rectangle (2), $a_2 = 20 \times 120 = 2400 \text{ mm}^2$

Distance between G_2 and reference axis AA, $y_2 = \dfrac{120}{2} = 60$ mm

Distance between centroid G of the T-section and the reference axis AA,

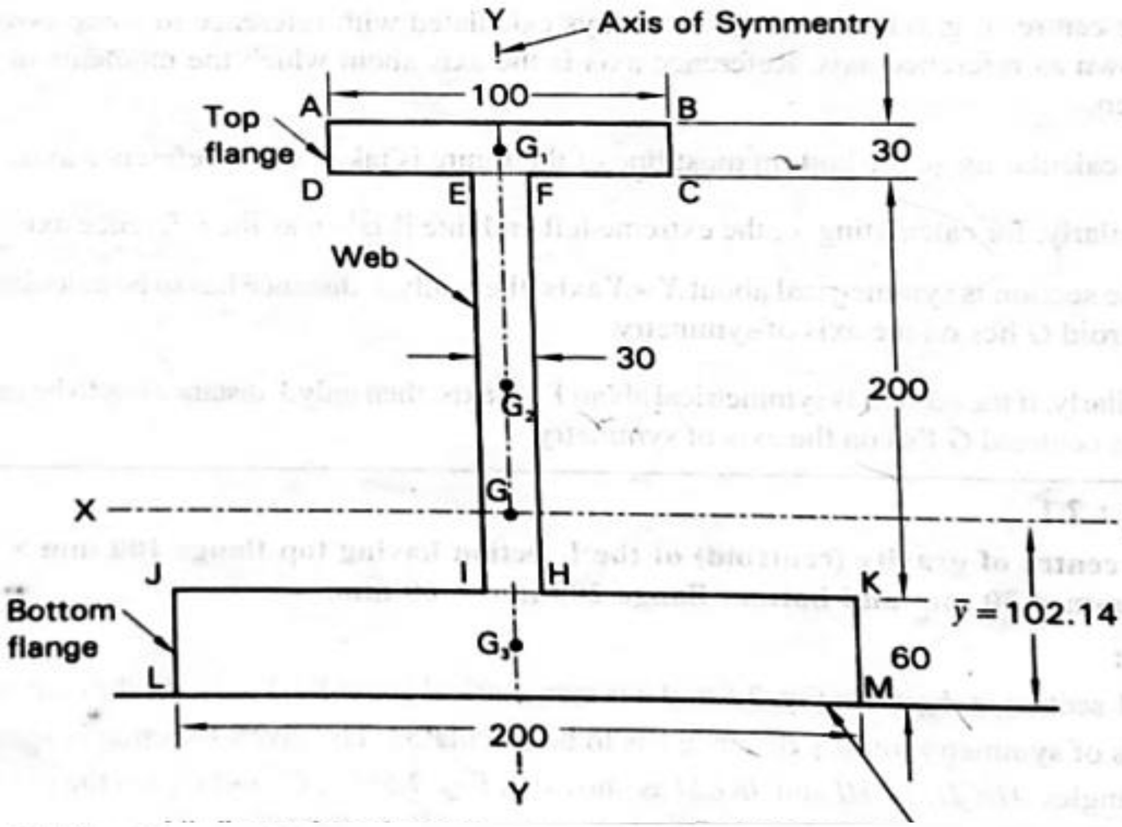
$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2400 \times 130) + (2400 \times 60)}{2400 + 2400} = 95 \text{ mm}$$

P4

Find the centre of gravity (centroid) of the I-section having top flange 100 mm × 30 mm, web 200 mm × 30 mm and bottom flange 200 mm × 60 mm.

Solution :

The I-section is shown in Fig. 2.5 and it is symmetrical about $Y-Y$ axis. As the centroid G lies on the axis of symmetry, only $\bar{y}$ distance has to be calculated. The given I-section is split up into three rectangles *ABCD*, *EFHI* and *JKLM* as shown in Fig. 2.5. G_1, G_2 and G_3 are the centroids of these three rectangles respectively. Let the bottom edge *ML* be the reference axis.



(i) Top flange ABCD

Area of rectangle *ABCD*, $a_1 = 100 \times 30 = 3000 \text{ mm}^2$

Distance between centroid G_1 of rectangle *ABCD* and reference axis *ML*,

$$y_1 = 60 + 200 + \frac{30}{2} = 275 \text{ mm}$$

(ii) Web EFHI

Area of rectangle *EFHI*, $a_2 = 200 \times 30 = 6000 \text{ mm}^2$

Distance between centroid G_2 of rectangle EFHI and reference axis *ML*,

$$y_2 = 60 + \frac{200}{2} = 160 \text{ mm}$$

(iii) Bottom flange JKLM

Area of rectangle *JKLM*, $a_3 = 200 \times 60 = 12000 \text{ mm}^2$

Distance between centroid G_3 of rectangle *JKLM* and reference axis *ML*,

$$y_3 = \frac{60}{2} = 30 \text{ mm}$$

Total area, $A = a_1 + a_2 + a_3 = 3000 + 6000 + 12000 = 21000 \text{ mm}^2$

$$\bar{y} = \frac{\text{Sum of moments of individual areas about reference axis } ML}{\text{Total area}}$$

$$= \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{A}$$

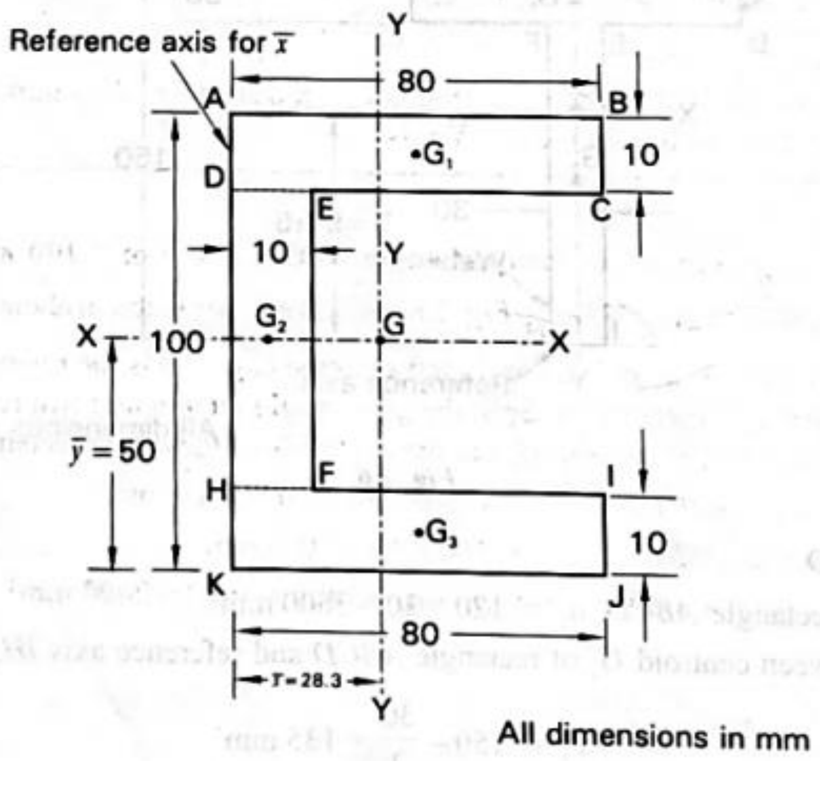
$$= \frac{(3000 \times 275) + (6000 \times 160) + (12000 \times 30)}{21000} = 102.14 \text{ mm}$$

Therefore centroid G of the I-section is at a distance of 102.14 mm from reference axis ML (i.e., bottom edge ML) and lies on the $Y - Y$ axis.

P5

Find the centre of gravity (centroid) of a channel section of size 100 mm × 80 mm × 10 mm.

Solution : The channel section is shown in Fig. 2.7 and it is symmetrical about $X - X$ axis. As the centroid G lies on the axis of symmetry, only $\bar{x}$ distance has to be calculated. The given channel section is split into three rectangles $ABCD$, DEFH and HIJK as shown in Fig. 2.7. G_1, G_2 and G_3 are the centroid of these three rectangles respectively. Let the left extreme edge AK be the reference axis.



(i) Rectangle ABCD

Area of rectangle $ABCD$, $a_1 = 80 \times 10 = 800 \text{ mm}^2$

Distance between centroid G_1 of rectangle $ABCD$ and reference axis AK,

$$x_1 = \frac{80}{2} = 40 \text{ mm}$$

(ii) Rectangle DEFH

Area of rectangle $DEFH$, $a_2 = (100 - 10 - 10) \times 10 = 800\ \text{mm}^2$

Distance between centroid G_2 of rectangle DEFH and reference axis AK,

$$x_2 = \frac{10}{2} = 5\ \text{mm}$$

(iii) Rectangle HIJK

Area of rectangle $HIJK$, $a_3 = 80 \times 10 = 800\ \text{mm}^2$

Distance between centroid G_3 of rectangle $HIJK$ and reference axis AK,

$$x_3 = \frac{80}{2} = 40\ \text{mm}$$

$$\text{Total area, } A = a_1 + a_2 + a_3 = 800 + 800 + 800 = 2400\ \text{mm}^2$$

$$\bar{x} = \frac{\text{Sum of moments of individual areas about reference axis } AK}{\text{Total area}} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{A}$$

$$= \frac{(800 \times 40) + (800 \times 5) + (800 \times 40)}{2400} = 28.3\ \text{mm}$$

Therefore centroid G of the channel section is at a distance of 28.3 mm from reference axis AK (i.e., left edge AK) and lies on the $X - X$ axis.

P6

Find the centre of gravity of an unequal angle section 150 mm × 100 mm × 10 mm.

Solution : The angle section is shown in Fig. 2.8 and it is not symmetrical about any axis. Therefore find out both the values of $\bar{x}$ and $\bar{y}$. For $\bar{x}$, left extreme edge AH is the reference axis and for $\bar{y}$, bottom edge HF is the reference axis. Split the given angle section into two rectangles $ABCD$ and $DEFH$ as shown in Fig. 2.8. G_1 and G_2 are the centroids of these two rectangles respectively.

Area of rectangle $ABCD$, $a_1 = (150 - 10) \times 10 = 1400\ \text{mm}^2$

Area of rectangle $DEFH$, $a_2 = 100 \times 10 = 1000\ \text{mm}^2$

For $\bar{x}$:

Distance between centroid G_1 of rectangle $ABCD$ and left edge AH,

$$x_1 = \frac{10}{5} = 5\ \text{mm}$$

Distance between centroid G_2 of rectangle DEFH and left edge AH,

$$x_2 = \frac{100}{2} = 50\ \text{mm}$$

Let $\bar{x}$ be the distance of centroid G of the angle section from left edge AH

$$\therefore \bar{x} = \frac{\text{Sum of moments of individual areas about reference axis } AH \text{ (i.e., left edge } AH)}{\text{Total area}}$$

$$= \frac{a_1x_1 + a_2x_2}{A} = \frac{(1400 \times 5) + (1000 \times 50)}{2400} = 23.75\ \text{mm}$$

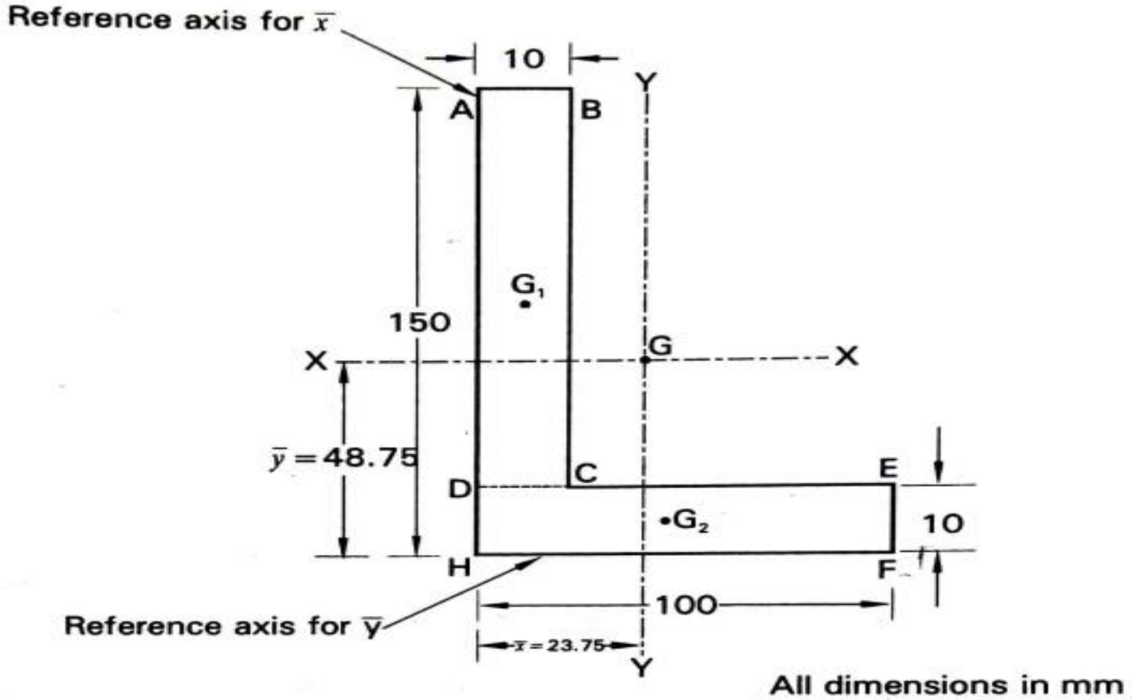


Fig. 2.8

Distance between centroid G_2 of rectangle *DEFH* and bottom edge *HF*,

$$y_2 = \frac{10}{2} = 5 \text{ mm}$$

Let $\bar{y}$ be the distance of centroid *G* of the angle section from bottom edge HF

$$\therefore \bar{y} = \frac{\text{Sum of moments of individual areas about reference axis } HF \text{ (i.e., bottom edge } HF)}{\text{Total area}}$$

$$= \frac{a_1y_1 + a_2y_2}{A} = \frac{(1400 \times 80) + (1000 \times 5)}{2400} = 48.75 \text{ mm}$$

Therefore centroid *G* of the angle section is 23.75 mm from left edge *AH* and 48.75 mm from bottom edge *HF*.

P7

Find the centroid of Lamina as shown in figure.

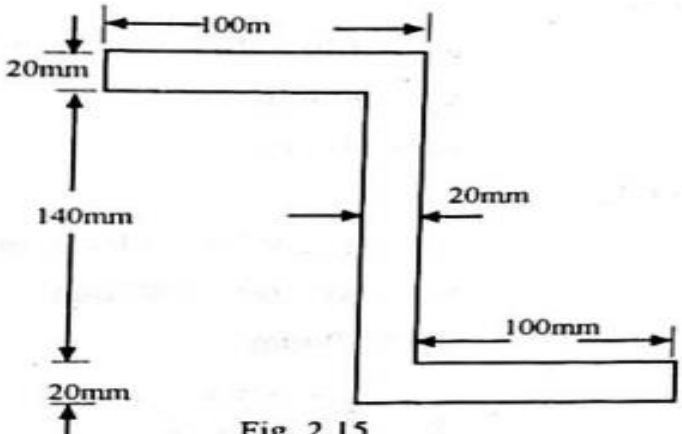


Fig. 2.15

Solution :

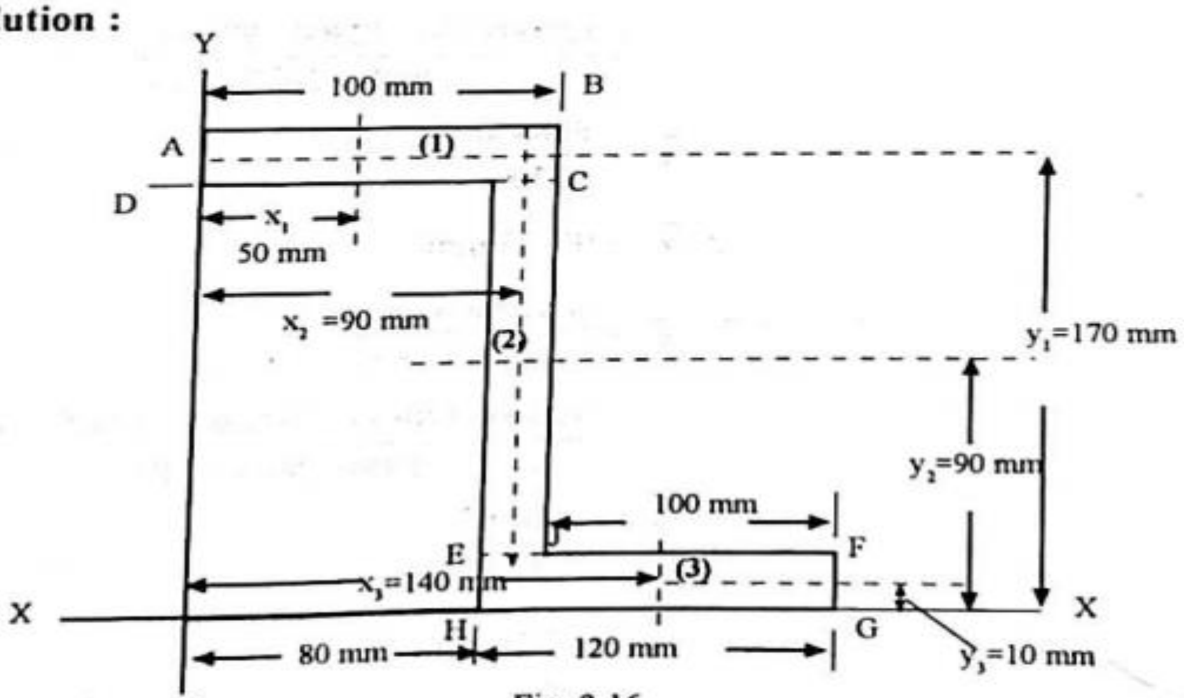


Fig. 2.16

As the section is unsymmetrical about any axis. Take the reference line x - x and y -y as shown in fig. 2.16

(i) Rectangle 1. (ABCD)

$$a_1 = 100 \times 20 = 2000 \text{ mm}^2$$
$$x_1 = 50 \text{ mm}$$
$$y_1 = 170 \text{ mm}$$

(ii) Rectangle 2. (CIEJ)

$$a_2 = 140 \times 20 = 2800 \text{ mm}^2$$
$$x_2 = 90 \text{ mm}$$
$$y_2 = 90 \text{ mm}$$

(iii) Rectangle 3. (EFGH)

$$a_3 = 120 \times 20 = 2400 \text{ mm}^2$$
$$x_3 = 80 + 60 = 140 \text{ mm}$$
$$y_3 = 10 \text{ mm}$$

$$\therefore \; \bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(2000\times50) + (2800\times90) + (2400\times140)}{2000 + 2800 + 2400}$$

$$\bar{x} = \frac{688000}{7200}$$

$$\therefore \; \bar{x} = 95.55 \text{ mm}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(2000 \times 170) + (2800 \times 90) + (2400 \times 10)}{2000 + 2800 + 2400}$$

$$\therefore \bar{y} = \frac{616000}{7200}$$

$$= 85.55 \text{ mm}$$

$\therefore$ Centroid of a section is $\bar{x} = 95.55$ mm and $\bar{y} = 85.55$ mm **Ans.**

$x_2 = 70$ mm

$y_2 = 70$ mm

(iii) Rectangle 3. (EFGH)

$a_3 = 80 \times 20 = 1600 \text{ mm}^2$

$x_3 = 100$ mm

$y_3 = 10$ mm

$$\therefore \bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(1600 \times 40) + (2000 \times 70) + (1600 \times 100)}{1600 + 2000 + 1600}$$

$$= \frac{364000}{5200}$$

$\therefore \bar{x} = 70$ mm **Ans.**

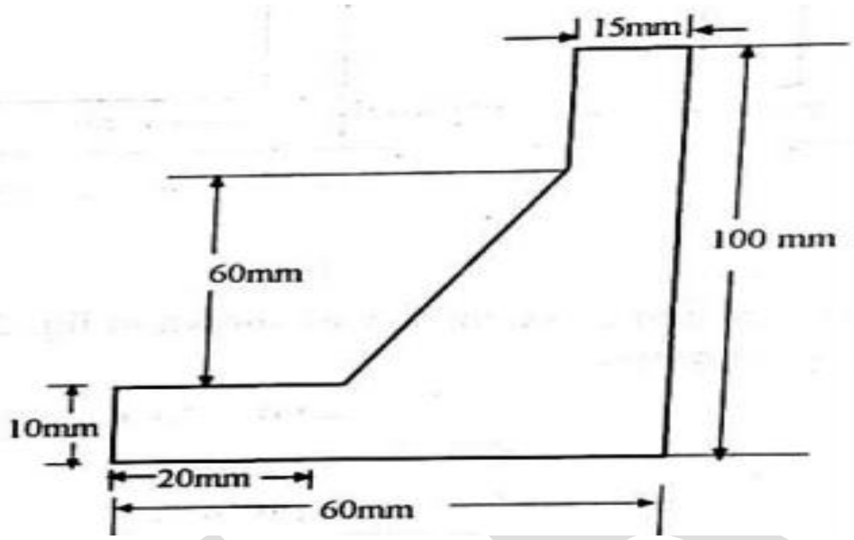
$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(1600 \times 130) + (2000 \times 70) + (1600 \times 10)}{1600 + 2000 + 1600}$$

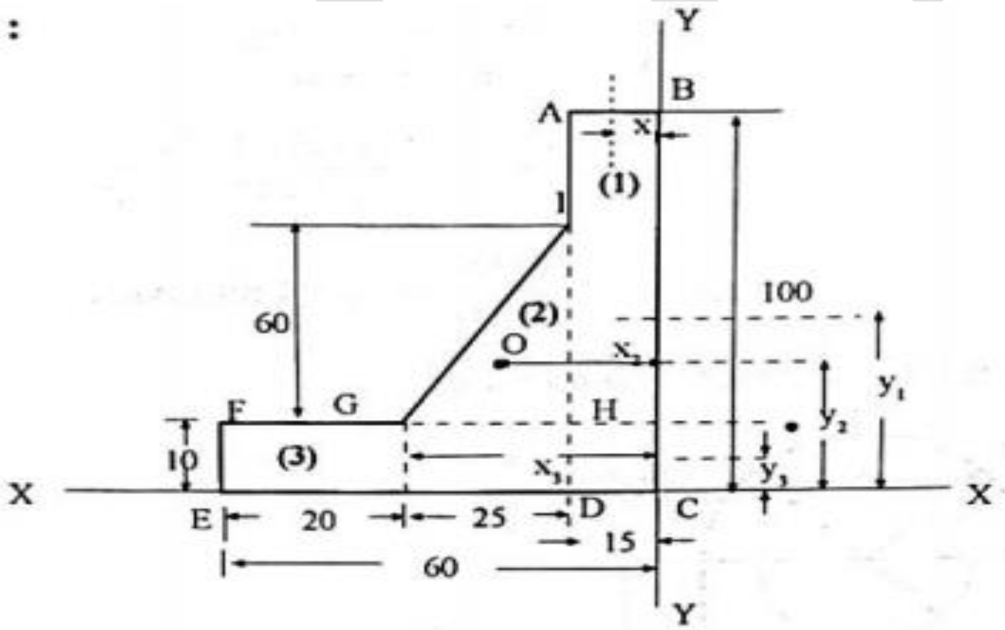
$$= \frac{364000}{5200}$$

$\therefore \bar{y} = 70$ mm **Ans.**

P 8 Find the centroid of given lamina.



Solution :



Taking the reference line x-x and y-y as shown in fig.2.20 and divide the lamina as ABCD, DEFH (Rectangles) and GHI (Triangle).

(i) Rectangle 1.

$a_1 = 100 \times 15 = 1500\ mm^2$

$x_1 = 15/2 = 7.5$ mm

$y_1 = 50$ mm

(ii) Triangle 2.

$a_2 = 1/2 \times 25 \times 60 = 750\ mm^2$ $\frac{1}{2} \times b \times h$

$x_2 = 15 + (25/3) = 23.33$ mm

$y_2 = 10 + \frac{60}{3} = 30$ mm

(iii) Rectangle 3.

$a_3 = 45 \times 10 = 450\ mm^2$

$x_3 = 15 + \frac{45}{2} = 37.5$ mm

$y_3 = 5$ mm

$$\therefore \quad \bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(1500\times7.5) + (750\times23.33) + (450\times37.5)}{1500 + 750 + 450}$$

$$\therefore \quad \bar{x} = \frac{45622.5}{2700}$$

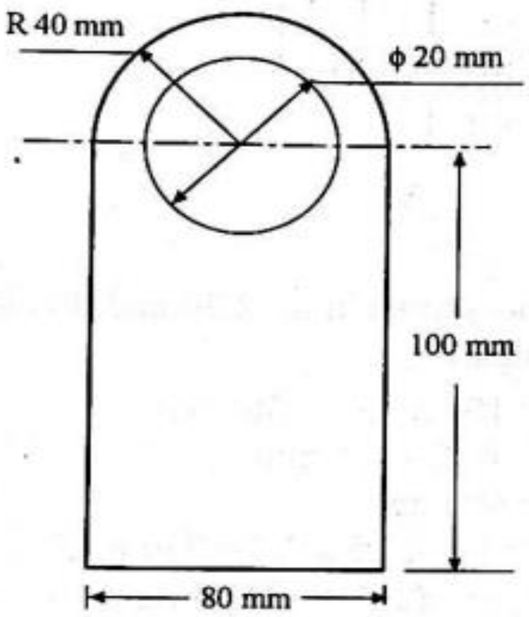
$$= 16.89 \text{ mm} \quad \textbf{Ans.}$$

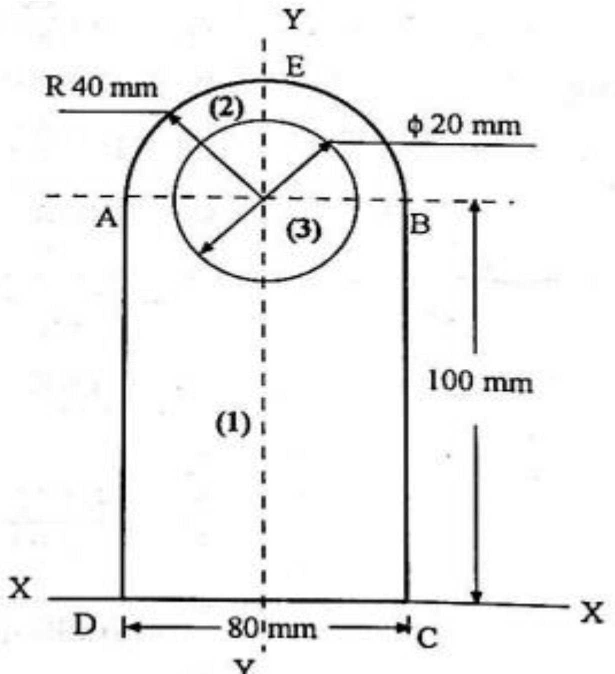
$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(1500\times50) + (750\times30) + (450\times5)}{1500 + 750 + 450}$$

$$\therefore \quad \bar{y} = \frac{99750}{2700} = 36.94 \text{ mm} \quad \textbf{Ans.}$$

Problem 2.9. *Find the centroid of a lamina.*





The lamina is symmetrical about y-axis, therefore, the reference lines are x-x and y-y as shown in fig.2.21 and $\bar{x} = 0$. Divide the fig. into (1) Rectangle ABCD (2) Semicircle AEB and (3) Circle.

1) Rectangle ABCD

$$a_1 = 100 \times 80 = 8000 \text{ mm}^2$$
$$y_1 = 50 \text{ mm}$$

2) Semicircle AEB

$$\frac{\pi d^2}{2} \rightarrow a_2 = 1/2\, \pi \times 40^2 = 2513.17 \text{ mm}$$
$$y_2 = 100 + \frac{(4 \times 40)}{3\pi} = 116.97 \text{ mm}$$

$\frac{4r}{3\pi}$

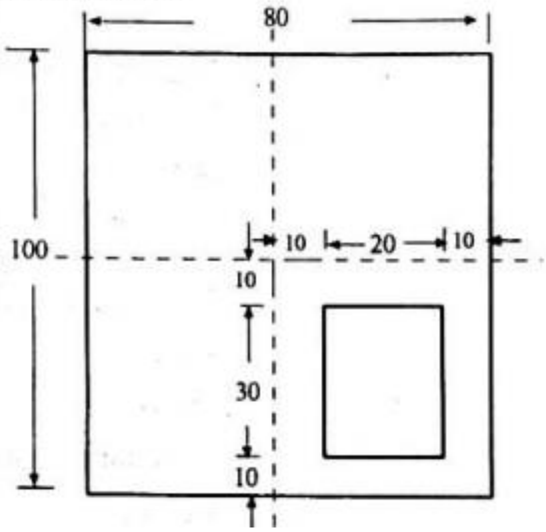
3) Circle

$$\frac{\pi d^2}{4} \rightarrow a_3 = \frac{\pi \times 20^2}{4} = 314.16 \text{ mm}^2$$
$$y_3 = 100 \text{ mm}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 - a_3y_3}{a_1 + a_2 - a_3}$$
$$= \frac{(8000 \times 50) + (2513.17 \times 116.97) - (314.16 \times 100)}{8000 + 2513.17 - 314.16}$$
$$\therefore \bar{y} = \frac{662549.50}{10199.01}$$
$$\therefore \bar{y} = 64.96 \text{ mm} \textbf{ Ans.}$$

P 10 Find the centroid of plane figure as shown in figure.

All dimensions are in mm



The plane is not symmetrical about any axis, and the reference lines are shown above. The plane is divided into 1) Rectangle ABCD and 2) Rectangle EFGH.

1) Rectangle

$$a_1 = 80 \times 100 = 8000 \text{ mm}^2$$
$$x_1 = 40 \text{ mm}$$
$$y_1 = 50 \text{ mm}$$

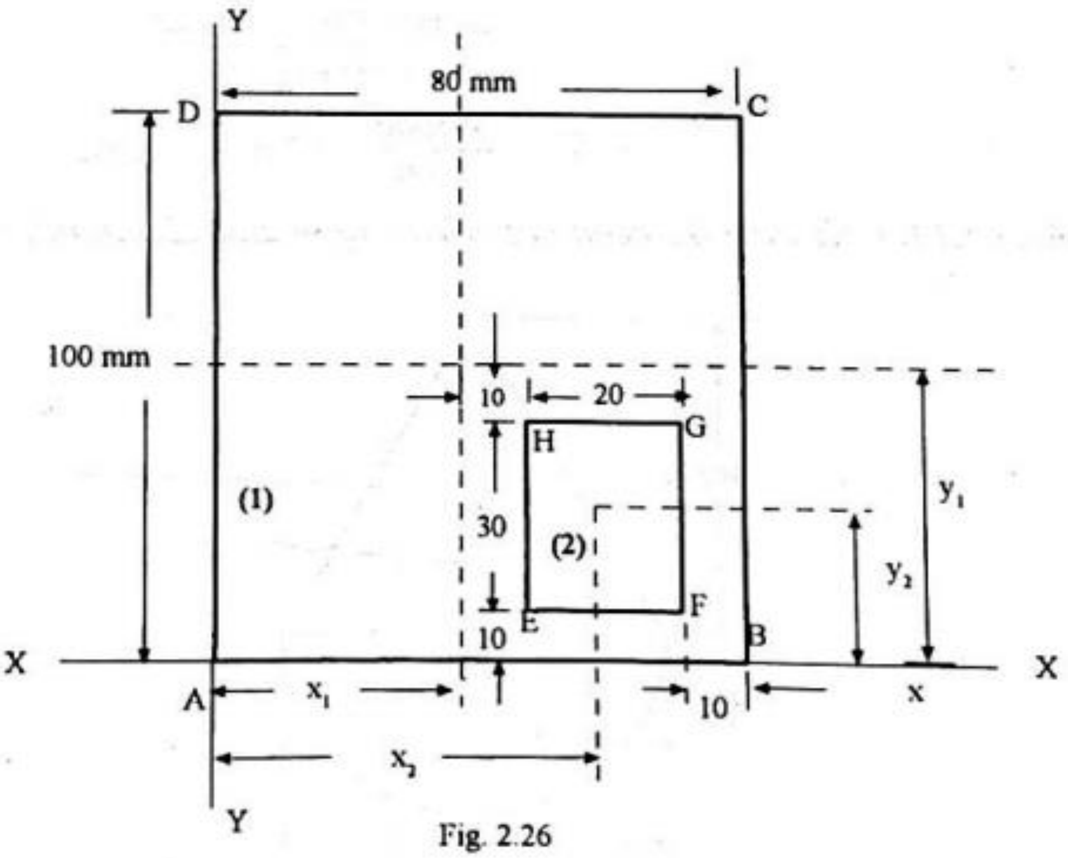


Fig. 2.26

2) Rectangle

$$a_2 = 30 \times 20 = 600 \text{ mm}^2$$

$$x_2 = 50 + 20/2 = 60 \text{ mm}$$

$$y_2 = 10 + 30/2 = 25 \text{ mm}$$

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{(a_1 - a_2)}$$

$$= \frac{(8000 \times 40) - (600 \times 60)}{(8000 - 600)}$$

$$= \frac{284000}{7400}$$

$$\therefore \bar{x} = 38.38 \text{ mm} \quad \textbf{Ans.}$$

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{(a_1 - a_2)}$$

$$= \frac{(8000 \times 50) - (600 \times 25)}{(8000 - 600)}$$

$$\therefore \bar{y} = \frac{385000}{7400} = 52 \text{ mm} \quad \textbf{Ans.}$$

P 11 Find the centroid of plane figure as shown in figure.

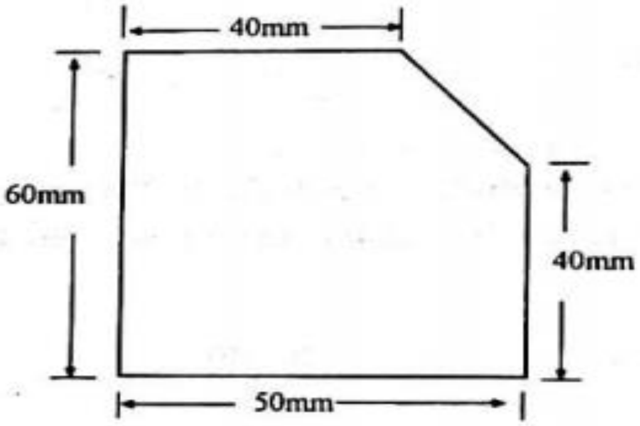
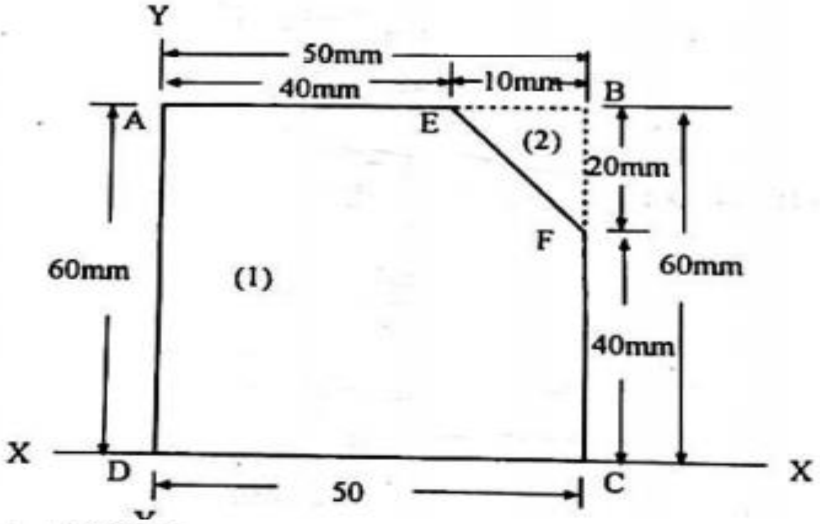


Fig. 2.29

Solution :



1. Rectangle ABCD :

$$\text{Area } a_1 = 60 \times 50 = 3000 \text{ mm}^2$$

$$x_1 = 25 \text{ mm}$$

$$y_1 = 30 \text{ mm}$$

2. Triangle BEF :

$$a_2 = \frac{1}{2} b\,h$$

$$= \frac{1}{2} 10 \times 20$$

$$= 100 \text{ mm}^2$$

$$x_2 = 40 + \left(\frac{10}{3}\right)$$

$$= 43.33 \text{ mm}$$

$$y_2 = 60 + \left(\frac{20}{3}\right)$$

$$= 66.66 \text{ mm}$$

$$\therefore \ \bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2}$$

$$= \frac{(3000 \times 25) - (100 \times 43.33)}{3000 - 100}$$

$$= 24.36 \text{ mm} \qquad \textbf{Ans.}$$

$$\therefore \ \bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2}$$

$$= \frac{(3000 \times 30) - (100 \times 66.66)}{3000 - 100}$$

$$= 28.73 \text{ mm} \qquad \textbf{Ans.}$$

P 12 Find the centroid of plane figure as shown in figure.

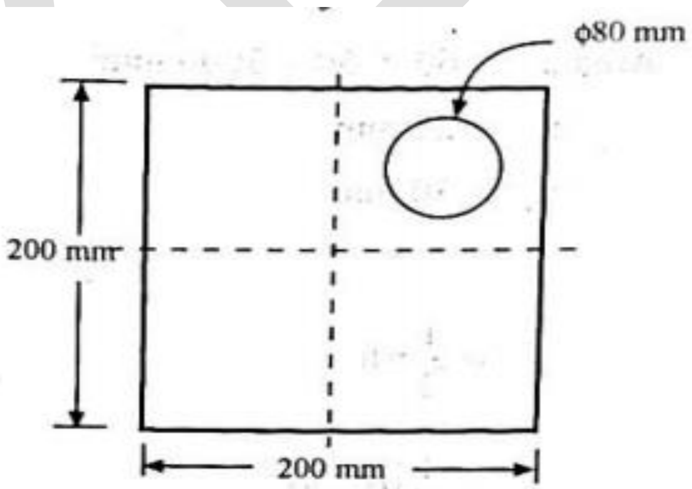
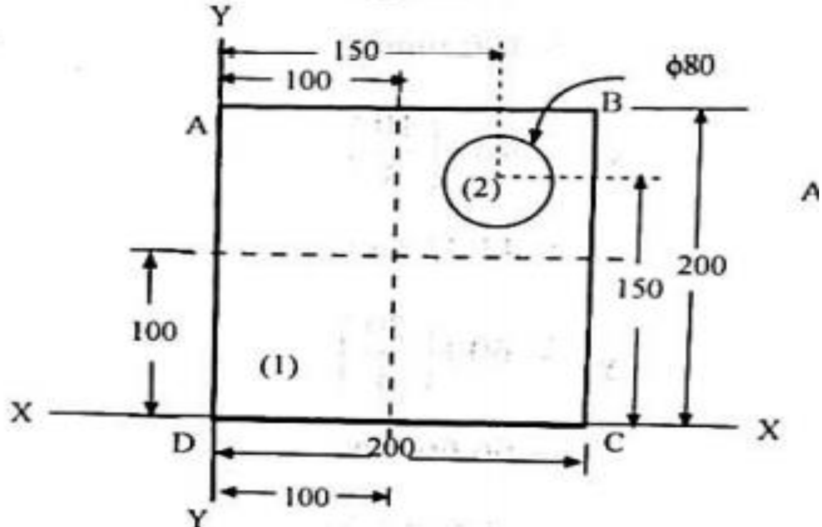


Fig. 2.31

Solution :



Let us divide the given section into (i) square ABCD and (ii) A circle

i) Square ABCD :

$$a_1 = 200 \times 200$$

$$= 40000 \text{ mm}^2$$

$$x_1 = 100 \text{ mm}$$

$$y_1 = 100 \text{ mm}$$

ii) Circle :

$$a_2 = \frac{\pi d^2}{4} = \frac{\pi \times 80^2}{4}$$

$$= 5026.54 \text{ mm}^2$$

$$x_2 = 150 \text{ mm}$$

$$y_2 = 150 \text{ mm}$$

$$\therefore \ \bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

$$= \frac{(40000 \times 100) - (5026.54 \times 150)}{(40000 - 5026.54)}$$

$$= 92.81 \text{ mm} \quad \textbf{Ans.}$$

$$\therefore \ \bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

$$= \frac{(40000 \times 100) - (5026.54 \times 150)}{(40000 - 5026.54)}$$

$$= 92.81 \text{ mm} \quad \textbf{Ans.}$$

2.9 MOMENT OF INERTIA

Moment of area about an axis is the product of the area and the perpendicular distance between CG (centre of gravity) of the area and the axis. It is known as first moment of area. If the moment of area is again multiplied by the perpendicular distance between the CG of the area and the axis, then the quantity is known as moment of the moment of area or second moment of area. This second moment of area is known as moment of inertia about that axis.

Hence, moment of inertia of an area about an axis is the product of the area and the square of the perpendicular distance between the CG of the area and that axis. It is represented by I. Moment of inertia about $X-X$ axis is represented by I_{XX} and about $Y-Y$ axis is represented by I_{YY}. The units of moment of inertia depends upon the unit of area and its CG's perpendicular distance. If both are in metre, then the unit of moment of inertia is m^4 and if both are in millimetre, then the unit of moment of inertia is mm^4. Sometimes, instead of area, mass of a body or figure is taken into account.

2.10 MOMENT OF INERTIA OF A PLANE FIGURE OR LAMINA OR AREA

A plane figure or lamina of total area A is shown in Fig. 2.9.

Divide the total area A into number of small areas whose centroids are known as shown in Fig. 2.9. Let a_1, a_2, a_3, etc., be the small areas and g_1, g_2, g_3, etc., be their centroids respectively.

Consider one of these strips (i.e., first strip).

Let, a_1 = Area of this trip

g_1 = CG of area a_1

x_1 = Perpendicular distance between the CG of area a_1 and the axis OY

y_1 = Perpendicular distance between the CG of area a_1 and the axis OX

Now, Moment of inertia (MI) = Area × (Perpendicular distance between the CG of area and the given axis)^2

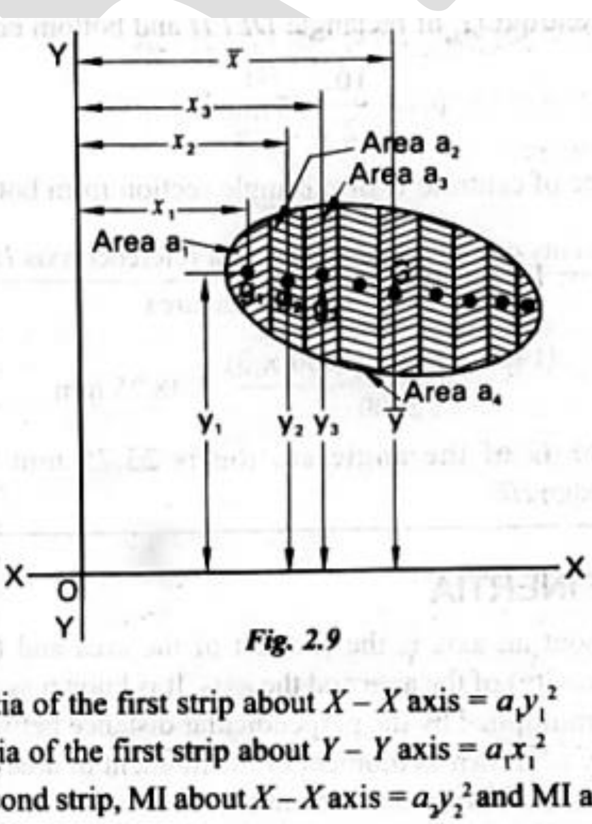


Fig. 2.9

∴ Moment of inertia of the first strip about $X-X$ axis $= a_1y_1^2$

Moment of inertia of the first strip about $Y-Y$ axis $= a_1x_1^2$

Similarly, for the second strip, MI about $X-X$ axis $= a_2y_2^2$ and MI about $Y-Y$ axis $= a_2x_2^2$ and so on.

∴ Moment of inertia of the whole area about $X-X$ axis

$$I_{XX} = a_1y_1^2 + a_2y_2^2 + a_3y_3^2 + ... = \sum ay^2$$

Also, $I_{XX} = \int y^2\, dA$, where dA = Area of the elemental strip

Moment of inertia of the whole area about $Y-Y$ axis,

$$I_{YY} = a_1x_1^2 + a_2x_2^2 + a_3x_3^2 + ... = \sum ax^2$$

Also, $I_{YY} = \int x^2\, dA$ where dA = Area of the elemental strip .

2.11 THEOREMS OF MOMENTS OF INERTIA

There are two theorems in moments of inertia. They are :

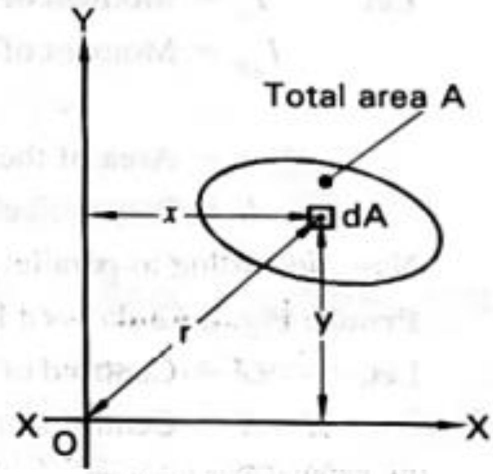
(i) Perpendicular axis theorem

(ii) Parallel axis theorem.

(i) Perpendicular Axis Theorem

Perpendicular axis theorem states, "The moment of inertia about an axis perpendicular to the plane and passing through the intersection of the other two axes $X-X$ and $Y-Y$ contained by the plane is equal to the sum of moments of inertia about $X-X$ and $Y-Y$ ".

i.e., If I_{XX} and I_{YY} be the moments of inertia of a plane section about two mutually perpendicular axes $X-X$ and $Y-Y$ intersecting at O in the plane of the section, then the moment of inertia of the section I_{ZZ} about the axis $Z-Z$, perpendicular or normal to the plane of section and passing through the intersection of $X-X$ and $Y-Y$ axes (i.e., passing through O) is given by



$$I_{ZZ} = I_{XX} + I_{YY}$$

(ii) Parallel Axis Theorem

Parallel axis theorem states, "The moment of inertia of a plane area with respect to any axis parallel to the centroidal axis in the plane of area is equal to the moment of inertia with respect to the centroidal axis plus the product of area of the figure and the square of the distance between the axes".

Let I_G = Moment of inertia of an area about centroidal axis

I_{AB} = Moment of inertia of the same area about an axis AB which is parallel to the centroidal axis

a = Area of the section

h = Perpendicular distance between the centroidal axis and the axis AB

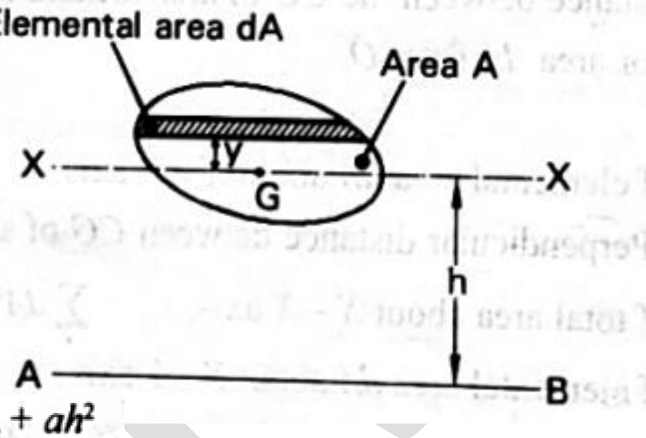
Let according to parallel axis theorem, $I_{AB} = I_G + ah^2$

$X-X$ = Centroidal axis in the plane of lamina

AB = An axis in the plane of lamina and parallel to centroidal axis $X-X$

h = Perpendicular distance between the centroidal axis $X-X$ and the axis AB

A = Total area of the lamina



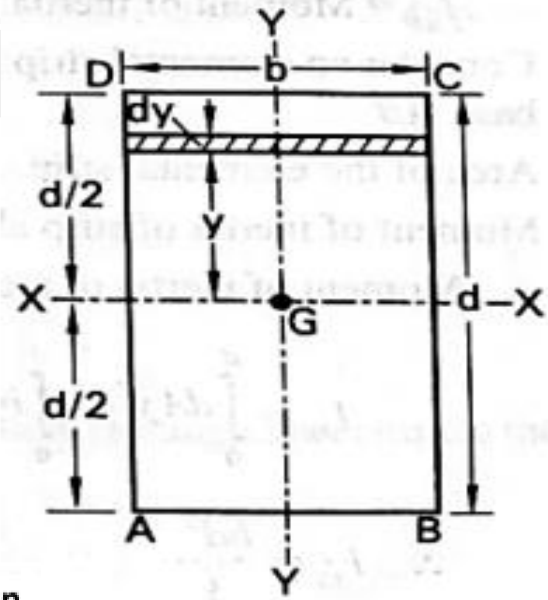
according to parallel axis theorem, $I_{AB} = I_G + ah^2$

2.12 MOMENT OF INERTIA OF A RECTANGULAR SECTION

(i) Moment of inertia of rectangular section about an axis $X-X$ passing through CG of the section

$$\therefore I_{XX} = \frac{bd^3}{12}$$

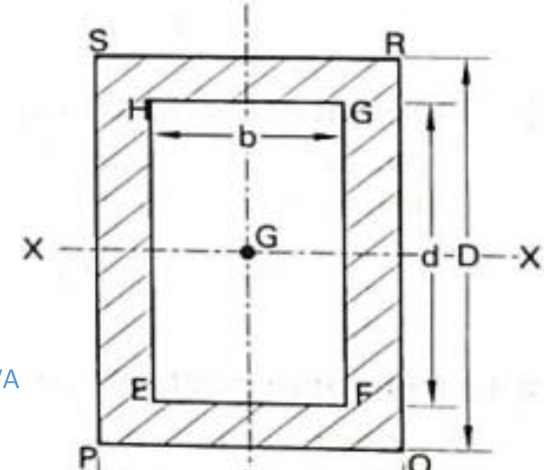
$$\therefore I_{YY} = \frac{db^3}{12}$$



Moment of inertia of a hollow rectangular section

$$I_{XX} = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{1}{12} \times (BD^3 - bd^3)$$

$$\text{Similarly, } I_{YY} = \frac{DB^3}{12} - \frac{db^3}{12} = \frac{1}{12} \times (DB^3 - db^3)$$



2.15 MOMENT OF INERTIA ABOUT CG FOR I-SECTION, T-SECTION, L-SECTION AND CHANNEL SECTION

The moment of inertia about CG for I-section, T-section, L-section and channel sections may be found out as per the following steps.

(i) Split up the given section into simple plane areas (i.e., rectangles).

(ii) Find the CG of the section.

(iii) Find the MI of each plane area (i.e., rectangular plane) about their centres of gravity (i.e., about G_1, G_2, G_3 etc.).

(iv) Using parallel axis theorem, transfer all these moment of inertia about the CG of the given section. [i.e., about $X-X$ or $Y-Y$ axis passing through the CG of the given section).

i.e., $I_1 = I_{G1} + a_1h_1^2$; $I_2 = I_{G2} + a_2h_2^2$; $I_3 = I_{G3} + a_3h_3^2$; and so on.

I_1, I_2, I_3, etc. = MI of rectangular section 1, 2, 3, etc., about $X-X$ or $Y-Y$ axis passing through the CG of the given section.

I_{G1}, I_{G2}, I_{G3}, etc = MI of rectangular section 1, 2, 3, etc., about an axis passing through its centre of gravity (i.e., G_1, G_2, G_3, etc) and parallel to $X-X$ or $Y-Y$ axis

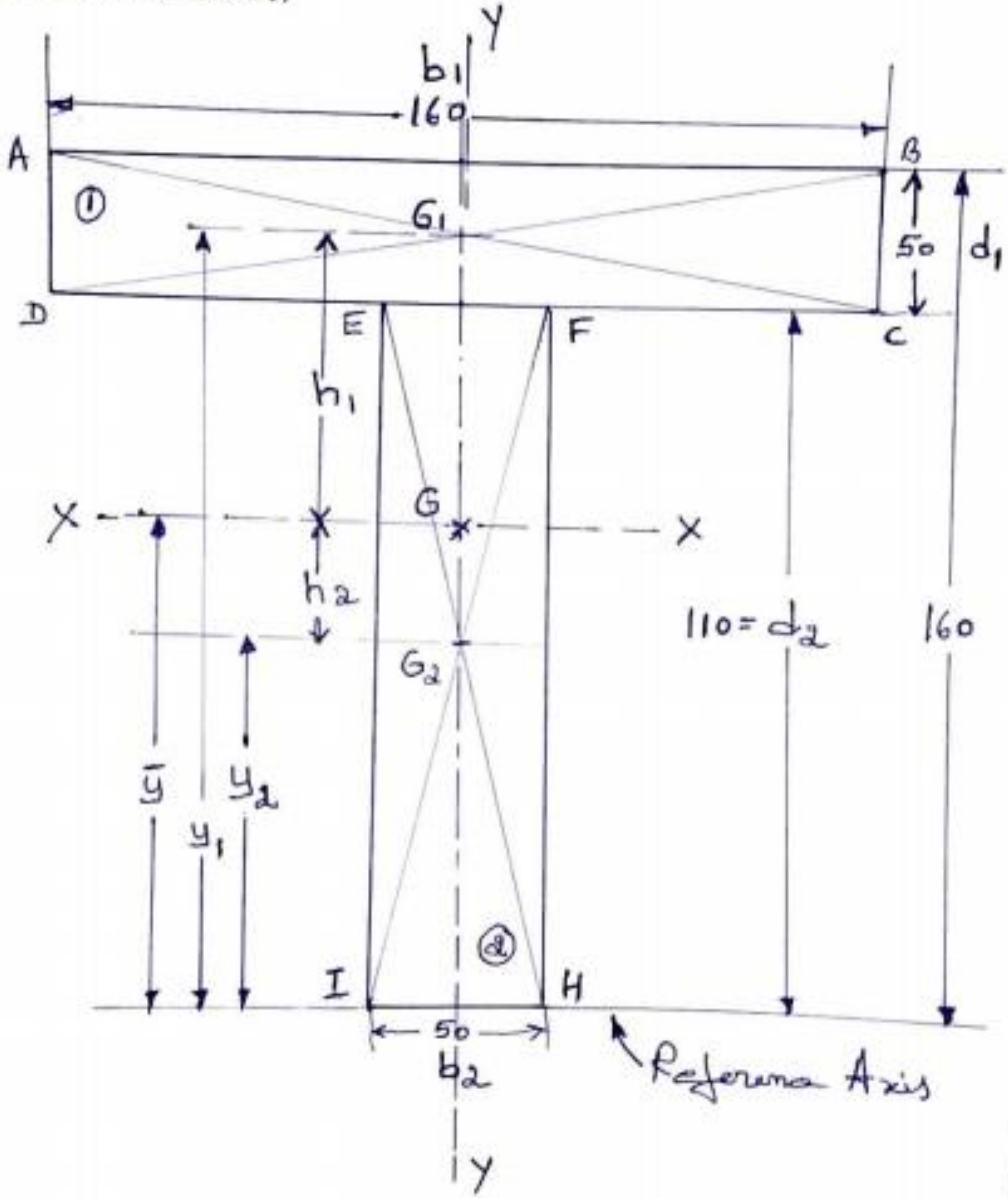
a_1, a_2, a_3, etc. = Area of rectangular sections 1, 2, 3, etc.

h_1, h_2, h_3, etc. = Perpendicular distance between the $X-X$ or $Y-Y$ axis and the centre of gravity of the rectangular section (i.e., G_1, G_2, G_3 etc.)

(v) Algebraic sum of the MI of the rectangular sections 1, 2, 3, etc., is the moment of inertia of the given section about $X-X$ or $Y-Y$ axis passing through CG.

i.e., $I = I_1 + I_2 + I_3 + \text{etc.}$; I is either I_{XX} or I_{YY}.

Find moment of Inertia about X-X and Y-Y axis of the T-Section 160 mm wide and 160 mm deep. The flange & Web thickness is 50 mm each.



T-Section is Symmetrical about Y-Y axis.

To find Centre of gravity.

$a_1 = 160 \times 50 = 8000\ mm^2 \quad y_1 = 160 - \frac{50}{2} = 135\ mm$

$a_2 = (160-50)\times 50 = 5500\ mm^2 \quad y_2 = \frac{160-50}{2} = 55\ mm$

$\therefore \bar{y} = \frac{a_1x_1 + a_2x_2}{a_1+a_2} = \frac{(8000\times 135)+(5500\times 55)}{8000+5500}$

$\boxed{\bar{y} = 102.4\ mm}$

To Find moment of Inertia about X-X axis.

$h_1 = y_1 - \bar{y} = 135 - 102.4 = 32.6\ mm$

$h_2 = \bar{y} - y_2 = 102.4 - 55 = 47.4\ mm$

$I_{xx} = I_{xx_1} + I_{xx_2}$

$= M.I.\ of\ Sec-1 + M.I\ of\ Sec-2$

$= \left[\frac{b_1d_1^3}{12} + a_1h_1^2\right] + \left[\frac{b_2d_2^3}{12} + a_2h_2^2\right]$

$= \left[\frac{160\times 50^3}{12} + 8000\times 32.6^2\right] + \left[\frac{50\times 160^3}{12} + 5500\times 47.4^2\right]$

$= 10.169\times 10^6 + 17.903\times 10^6 = 28.072\times 10^6\ mm^4$

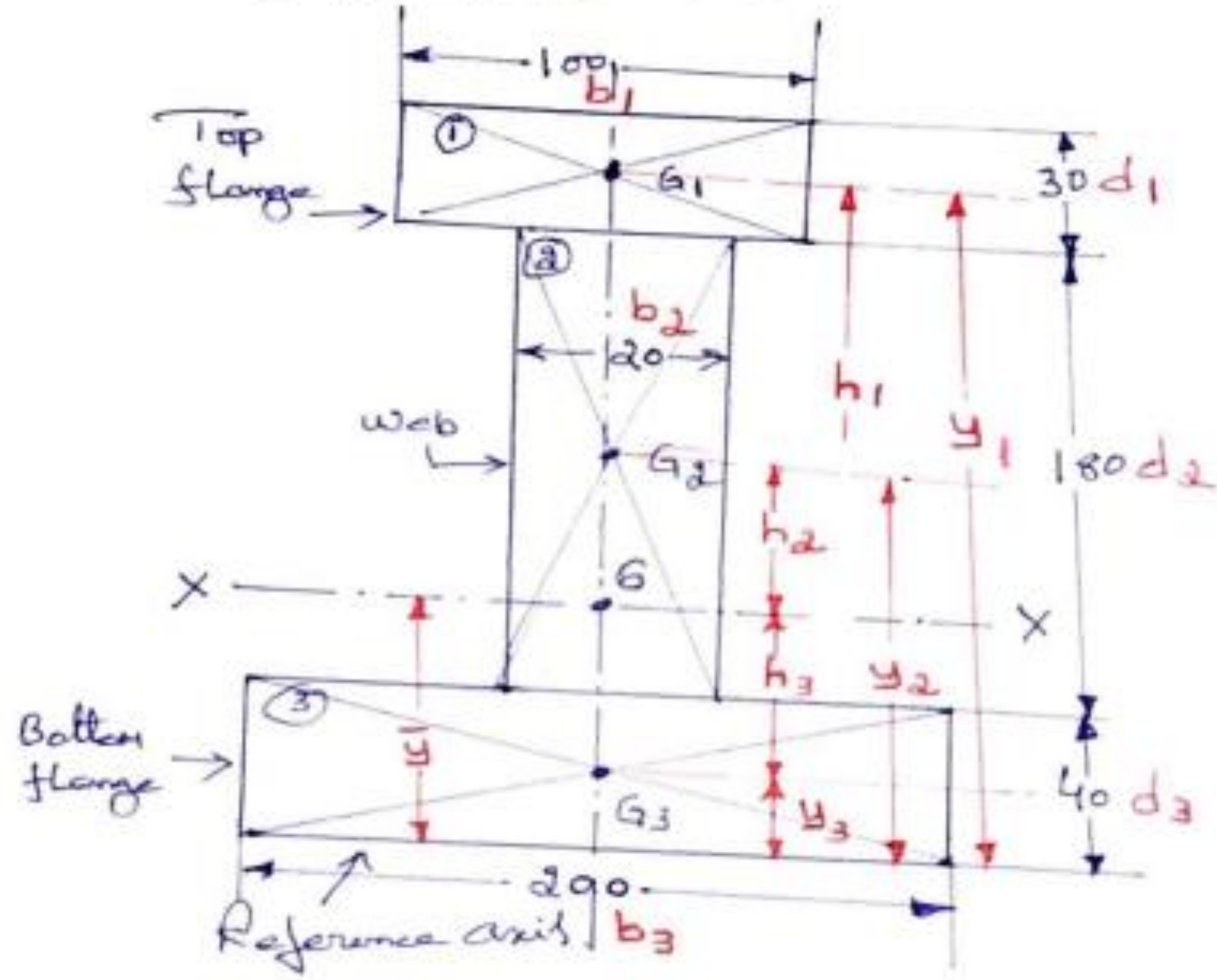
To find MI about Y-Y axis

$I_{yy} = I_{yy_1} + I_{yy_2} = \frac{d_1b_1^3}{12} + \frac{d_2b_2^3}{12} \quad \boxed{h_3 \,\&\, h_4 = 0}$

$= \frac{50\times 160^3}{12} + \frac{110\times 50^3}{12} = 17.0667\times 10^6 + 1.1458\times 10^6$

$I_{yy} = 18.2125\times 10^6\ mm^4$

Find the MI about axis passing through CG parallel and perpendicular to the base of an I-Section consist of top flange 100 mm × 30 mm, bottom flange 200 mm × 40 mm and web 180 × 20 mm.



$a_1 = 100 \times 30 = 3000\,mm^2$

$a_2 = 180 \times 20 = 3600\,mm^2$

$a_3 = 200 \times 40 = 8000\,mm^2$

$y_1 = 40 + 180 + \frac{30}{2} = 235\,mm$

$y_2 = 40 + \frac{180}{2} = 130\,mm$

$y_3 = \frac{40}{2} = 20\,mm$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = 91.3\,mm.$$

$h_1 = y_1 - \bar{y}$

$h_2 = y_2 - \bar{y}$

$h_3 = \bar{y} - y_3$

MI about X-X axis

$$I_{xx} = \left[I_{xx_1} + I_{xx_2} + I_{xx_3} \right]$$

$$= \left[\frac{b_1 d_1^3}{12} + a_1 h_1^2 \right] + \left[\frac{b_2 d_2^3}{12} + a_2 h_2^2 \right] + \left[\frac{b_3 d_3^3}{12} + a_3 h_3^2 \right]$$

$$=$$

$$= 62174070 + 15111684 + 41736186.67$$

$$= 119021940.7 = \underline{119.022 \times 10^6 \, mm^4}$$

MI about Y-Y axis.

$$I_{yy} = \left[I_{yy_1} + I_{yy_2} + I_{yy_3} \right]$$

$$= \left[\frac{d_1 b_1^3}{12} + a_1 h_4^2 \right] + \left[\frac{d_2 b_2^3}{12} + a_2 h_5^2 \right] + \left[\frac{d_3 b_3^3}{12} + a_3 h_6^2 \right].$$

$h_4; h_5, h_6 = 0$

$$= \left[\frac{d_1 b_1^3}{12} \right] + \left[\frac{d_2 b_2^3}{12} \right] + \left[\frac{d_3 b_3^3}{12} \right]$$

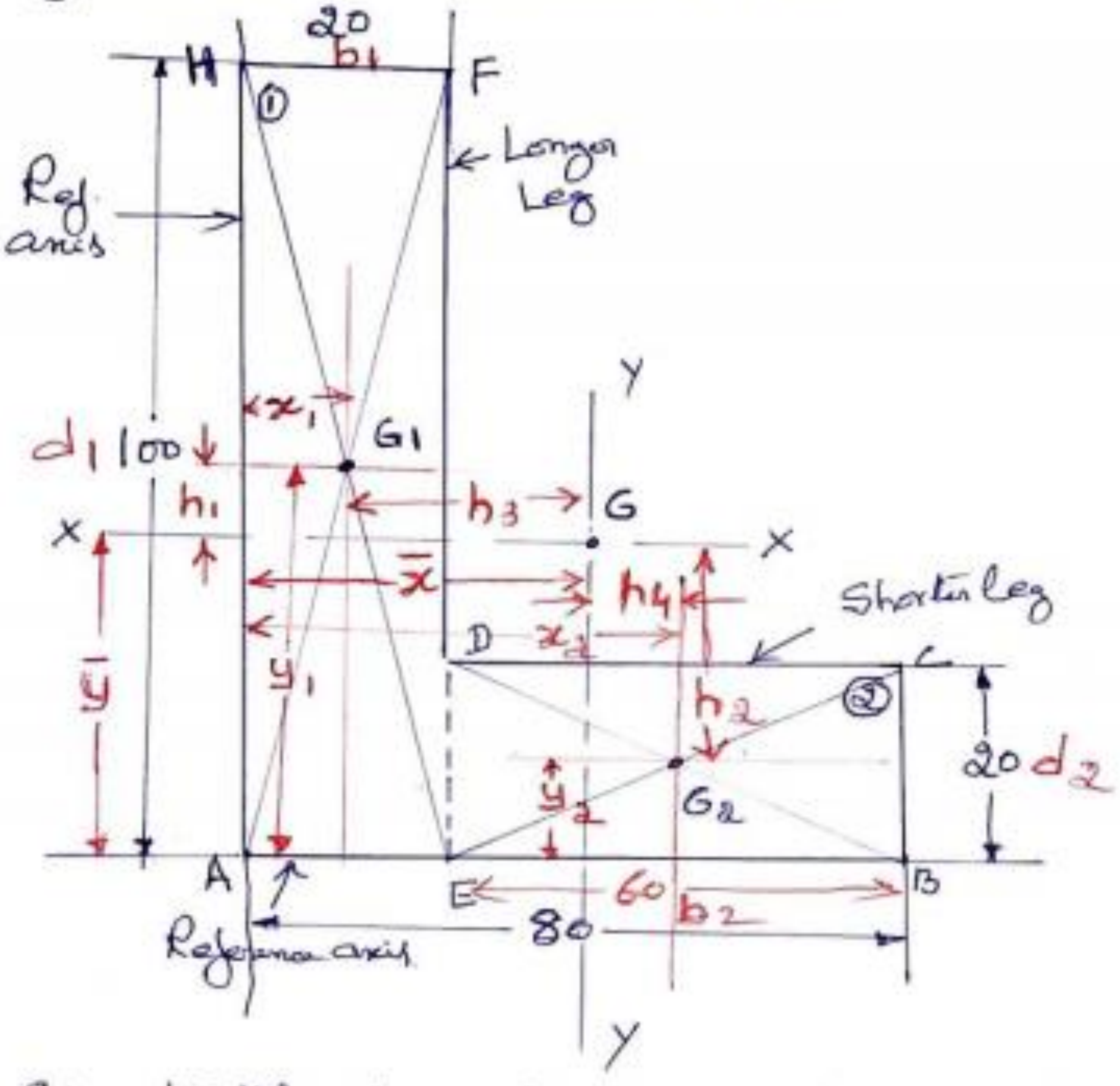
$$=$$

$$= 2500000 + 120000 + 26666666.67$$

$$= 29286666.67$$

$$= 29.287 \times 10^6 \, mm^4$$

Find MI about an axis passing through CG, parallel & perpendicular to Shorter Leg of an angle Section of dimensions 100 X 80 X 20 MM.



$a_1 = 100 \times 20 = 2000 mm^2$

$a_2 = (80-20) \times 20 = 1200 mm^2$

$h_1 = y_1 - \bar{y}$

$h_2 = \bar{y} - y_2$

$h_3 = \bar{x} - x_1$

$h_4 = x_2 - \bar{x}$

$y_1 = \frac{100}{2} = 50 MM$

$y_2 = \frac{20}{2} = 10 MM.$

$x_1 = \frac{20}{2} = 10 mm$

$x_2 = 20 + \frac{60}{2} = 50 MM$

$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = 25 MM$

$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = 35 MM$

I. M.I about X-X axis.

$h_1 = 15 mm$
$h_2 = 25 mm$

$$I_{xx} = I_{xx_1} + I_{xx_2}$$

$$= \left[\frac{b_1 d_1^3}{12} + a_1 h_1^2\right] + \left[\frac{b_2 d_2^3}{12} + a_2 h_2^2\right]$$

$$=$$

$$= 2116666.667 + 790000$$
$$= 2906666.667$$
$$= \underline{2.9067 \times 10^6 \ mm^4}$$

II M.I about Y-Y axis

$h_3 = 15 mm$
$h_4 = 25 mm$

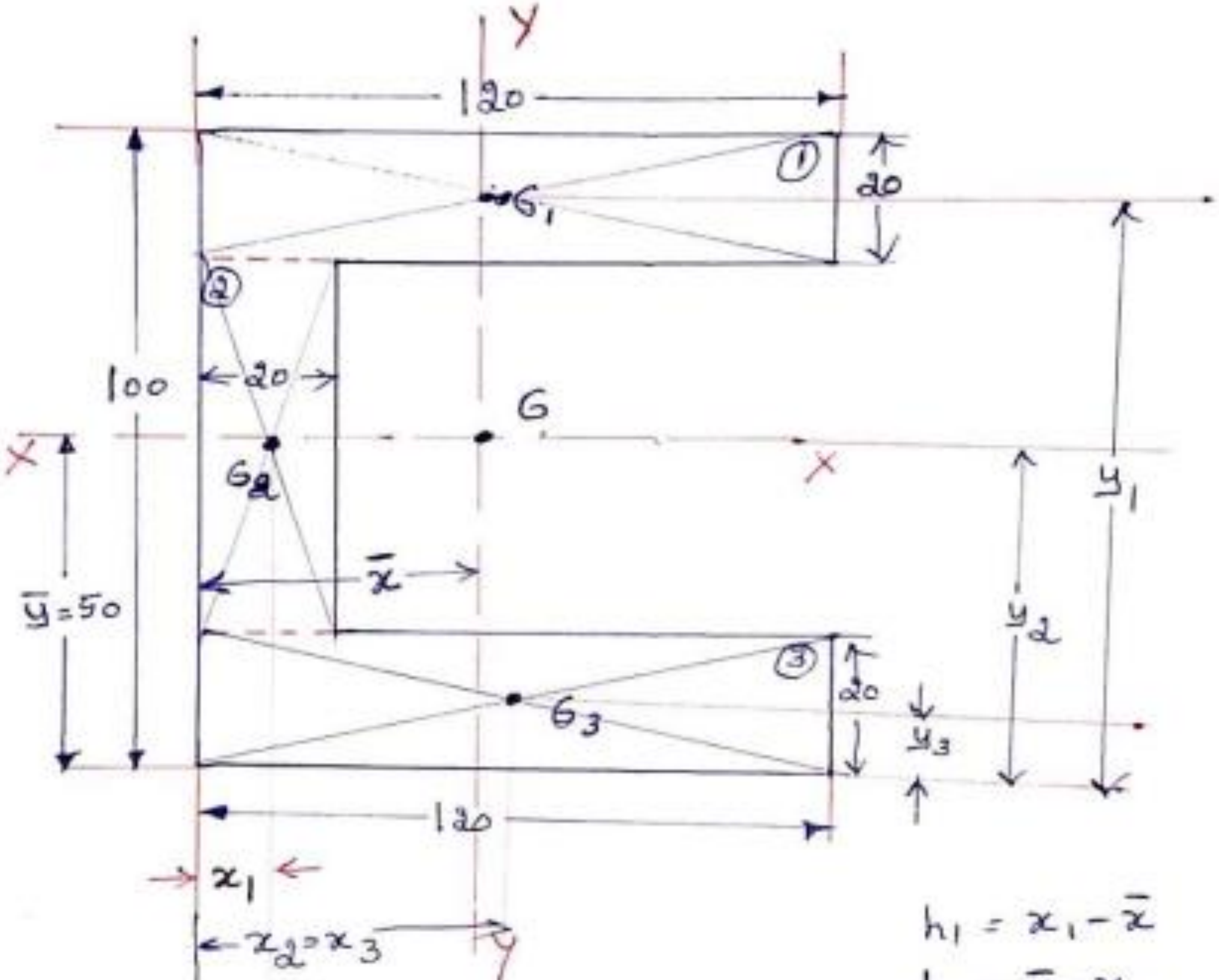
$$I_{yy} = I_{yy_1} + I_{yy_2}$$

$$= \left[\frac{d_1 b_1^3}{12} + a_1 h_3^2\right] + \left[\frac{d_2 b_2^3}{12} + a_2 h_4^2\right]$$

$$=$$

$$= 516666.667 + 1100000$$
$$= 1626666.667$$
$$= \underline{1.627 \times 10^6 \ mm^4}$$

Determine MI of the channel section of size 100 x 120 x 20 mm about X-X & Y-Y axis.



$h_1 = x_1 - \bar{x}$

$h_2 = \bar{x} - x_2$

$h_3 = x_3 - \bar{x}$

$a_1 = 120 \times 20 = 2400 \text{ mm}^2$

$a_2 = 60 \times 20 = 1200 \text{ mm}^2$

$a_3 = 120 \times 20 = 2400 \text{ mm}^2$

$x_1 = \frac{120}{2} = 60 \text{ mm}$ $\quad y_1 = 20 + 60 + \frac{20}{2} = 90 \text{ mm}$

$x_2 = \frac{20}{2} = 10 \text{ mm}$ $\quad y_2 = 20 + \frac{60}{2} = 50 \text{ mm}$

$x_3 = \frac{120}{2} = 60 \text{ mm}$ $\quad y_3 = \frac{20}{2} = 10 \text{ mm}$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} = 50 \text{ mm}$$

M.I. about X-X axis.

$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{120 \times 100^3}{12} - \frac{100 \times 60^3}{12} = 8.2 \times 10^6 \text{ mm}^4$$

Considering as hollow rectangular Section

i.e. $B = 120$ mm $D = 100$ mm $b = 100$ mm $d = 60$ mm

M.I. about Y-Y axis.

$d_1 = 20$ $b_1 = 120$
$d_2 = 60$ $b_2 = 20$
$d_3 = 20$ $b_3 = 120$

$$I_{yy} = I_{yy1} + I_{yy2} + I_{yy3}$$

$$= \left[\frac{d_1 b_1^3}{12} + a_1 h_1^2\right] + \left[\frac{d_2 b_2^3}{12} + a_2 h_2^2\right] + \left[\frac{d_3 b_3^3}{12} + a_3 h_3^2\right]$$

$=$

$$= 8.2 \times 10^6 \text{ mm}^4$$

ASSIGNMENT – 2

Q 1. Define centre of gravity, centroid.

Q 2. List the methods of finding Centre of gravity of simple figures.

Q 3. Locate the CG FOR Rectangle, Square, Triangle, Circle,Semicircle, Trapezium, Cone and hemisphere.

Q 4. Define Moment of Inertia.

Q 5, State Parallel Axis Theorem and Perpendicular Axis Theorem.

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All the solved problems from this chapter.